Introduction to Stochastic Processes

REMINDERS:

RANDOM VARIBLE

This is a special type of variable which assigns values to sets, every value is associated with a "CHANCE TO BE ASSIGNED". Let it be clarified that a random variable does not assign probabilities, the probability is already assigned with the function P from the triple (.

Let

**:**

* {
* (

EXPLANATION:

A **stochastic process**, also known as a **random process** is a set of random variables guided by a parameter, the parameter can be anything but is usually used for **TIME**.

A stochastic process can be either **DISCRETE** or **CONTINUOUS** and is the opposite of a deterministic process, any random values can be assigned to the process with a probability instead of a fixed value stream.

DEFINTION:

Stochastic / Random processes are denoted:

* {

Where :

* For a discrete process, T = N
* For a continuous process, T =

EXAMPLES:

1. If the value of the dollar changes every day Then,

{

1. If the price of a stock / future / option changes continuously Then,

{

Identical Stochastic Processes

EXPLANATION:

It is **almost-impossible** that there exists a time that when "activating" two stochastic processes on the same sample dot they will return different results.

Or

The possibility that a time t which returns different results for the same sample dot

DEFINTION:

Two stochastic processes

Or

*(NEED TO VERIFY THIS)*

EXAMPLE:

Let

|  |  |  |
| --- | --- | --- |
| t |  | ) |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

|  |  |  |
| --- | --- | --- |
| t |  | ) |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Similar Stochastic Processes

EXPLANATION:

It is **almost-impossible** that when "activating" two stochastic processes on the same sample dot using any time will return different results. At least one time will give the same result

Or

The possibility that every time returns different results for the same sample dot :

DEFINTION:

Two stochastic processes

*Or*

*(NEED TO VERIFY THIS)*

EXAMPLE:

Let

|  |  |  |
| --- | --- | --- |
| t |  | ) |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

|  |  |  |
| --- | --- | --- |
| t |  | ) |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

Every has at least one t which returns a different result!

* IMPORTANT NOTICE
* sided) stochastic processes are IDENTICAL if they are SIMILAR. (SIMILAR

Identical-Distribution Stochastic Processes

EXPLANATION:

Stochastic processes distribute the same if and only if the UNION of all their members distribute the same.

DEFINTION:

Two stochastic processes

Stochastic Processes - Independent Increments

EXPLANATION:

The difference between any two ordered pairs is independent of any other ordered pairs, therefore knowing the difference contributes no information to the difference

DEFINTION:

A stochastic process {

EXAMPLE:

* Assuming a "RANDOM / DRUNK WALK" the difference in step size over the last two steps has adds no information over the difference in the next two steps.

Stochastic Processes - Stationary Increments

EXPLANATION:

Adding a fixed amount of time h to a difference between random variables in a process does not change the distribution.

DEFINTION:

A stochastic process {

Or

Or

Conditioned Expectation

REMINDERS:

EXPECTED VALUE  
Let , Y be random variables:

Discrete Variables

E [] =   
E [] =   
E [] =

Continuous Variables

E [] =   
E [] =   
E [] =

General Properties

E [] =   
E [ +] = E [] +   
E [] = E []  
E [ + ] =

E [] = E []  
E [] =

Advanced Properties

]] =   
]] = E []

All proofs will be found on file – "Introduction to Probability Theory" including information and examples for the uses of the expected value of a random variable

Expectation Conditioned by an Event

Let

Conditioning an expected value with OMEGA

**= =**

Conditioning an expected value with another event

**= =**

EXAMPLE:

,

,

= =

Expectation Conditioned by a Random Variable

Let

is a function of , its values are dependent on what happens with .

It is sometimes denoted which shows it is has a value of each instance of .

EXAMPLE:

GAME -

Throwing three coins with numbers on their heads, then you gain points equal to the sum of the coins that landed with HEAD's up.

Assume three coins with the following values on their head:

* Coin 1: 10
* Coin 2: 20
* Coin 3: 50

,

= 40

=

Expectation Conditioned by a

This idea is based on the assumption that E[X|Y] relies only on the minimal sigma-field generated by Y, . Therefore, conditioning by Y is equal to condition by

Let us then condition by an arbitrary sigma field .

Let be random variables, G, H are sigma fields:

PROPERTIES:

1. is monotone and linear

Stochastic Process – Filtration

We assume that as time goes on we accumulate more information about our processes.

As time t increases, so does our knowledge about what has happened in the past, this is called Filtration.

DEFINTION:

,

And

EXAMPLE:

At time t = 4 it will be possible to decide whether or not A has occurred, therefore, A

Names:

*Brownian Process*

*Definition 1:*

*Definition 2:*

*Definition 3:*

Properties

Proof:

Let

E [

1. Paul-Levy

and is Square-Integrable then

Examples

then prove that:



Names:

Skorokhod's Representation Theorem (משפט ההצגה של שקורוחוד)

*Definition:*

Let X be a random variable such that:

Then there exists a stopping time T and a Brownian motion such that

X ~

Names:

Wald's Equation

*Definition:*

be Independent, equally distributed random variables with MEAN = , T is a discrete stopping time then: